Kuwait University | Math. 101 | June 17, 1993
Mathematics Dept. | Final Examination | Duration: 2 hours

Questions 1-6 are weighted equally at 14 points each, question 7 is worth 16 points.
Calculators are not allowed.

1 (a) Evaluate $\lim_{x \to 0} \dfrac{\tan^2(5x)}{x^3 + x^2}$, if it exists.

(b) Suppose f is a function given by $f(x) = (1+5x)^{10}$, $x \in R$. Find $\lim_{x \to 0} \dfrac{f(x) - 1}{x}$.

2. (a) Let y be a function of x defined implicitly by $x \sin 2y = y \cos 2x$. Find the equation of the normal to the graph of y at $(\frac{\pi}{4}, \frac{\pi}{2})$.

(b) Is the following function continuous at $x = 0$? If not, explain why not.

$$f(x) = \begin{cases} \dfrac{\sqrt{1 + x \sin x} - \cos x}{x\,|x|} & , \text{if } x \neq 0 \\ 1 & , \text{if } x = 0 \end{cases}$$

3. (a) Let f be a function such that $f'(x) = 2$, $\forall x \in R$ and $f(0) = 5$. Find $f(7)$.

(b) Find the point (x, y) on $y + 3x = 3$ so that the area of the shaded rectangle is maximum. (See the figure below)

4 (a) Let $f(x) = x^2 (x-2)^{\frac{2}{3}}$, $x \in R$. Show that the graph of f has a cusp.

(b) Use differentials to find an approximate value of $\sqrt{2 + \sqrt{4.08}}$.

5. Evaluate the following integrals: (a) $\int_0^3 |x^2 + 2x - 3|\, dx$ (b) $\int \dfrac{\cos 5x}{(1 + \sin 5x)^4} dx$.

6. (a) Let $f(x) = \dfrac{x}{\sqrt{x+1}}$. Find the average value of f on $[0, 3]$.

(b) Show that the function $f(x) = \int_{\frac{1}{x}}^{\sqrt{x}} \sqrt{t^4 + 1}\, dt$ is increasing on $(0, \infty)$.

7. Let $f(x) = \dfrac{(x-2)}{x^2}$,

(a) Find the horizontal and vertical asymptotes of f (if any).

(b) Find the intervals where f is increasing and the intervals where f is decreasing.

(c) Find the local extrema of f.

(d) Find the intervals where the graph of f is concave upward, and the intervals where the graph of f is concave downward. Find the points of inflection (if any).

(e) Sketch the graph of f.